

Numerical Modeling of Under Saturated Oil Reservoir with Edge Water Influx

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Abstract

The governing equations that describe the physical processes occurring during primary depletion of an under saturated reservoir were formulated. A general fluid flow equation for the reservoir was derived by integrating the conservation of mass, Darcy's law, and the equation of state. The well and aquifer were modeled using Peaceman's well model and the Fetkovich unsteady-state aquifer model, respectively. The reservoir was discretized into a grid of blocks. Coupling of the reservoir, well, and aquifer was achieved implicitly through a finite difference method to develop a comprehensive three-dimensional reservoir-aquifer model. Simulations were conducted in MATLAB, utilizing MATLAB's Graphical User Interface (GUI) and programming environment. A time step of 30 days was employed to advance the simulation over a period of 4,000 days (approximately 11 years) of oil production. The results indicated that the reservoir's primary energy was exhausted when pressure dropped to 5,225 psi. An estimated oil recovery of 2.5 million barrels (MMbbl) was achieved, resulting in a recovery factor of 13.44%. The three-dimensional reservoir-aquifer model developed in this study provides a robust tool for predicting the performance of under saturated reservoirs bounded by edge water drive aquifers.

Introduction

Modeling of under saturated reservoirs bounded by active aquifers is a critical aspect of reservoir engineering. Understanding the behavior and dynamics of such reservoirs is crucial for optimizing production and maximizing recovery. Numerous theoretical studies have been conducted to develop mathematical models that accurately describe the behavior of under saturated reservoirs bounded by active aquifers (Khosravi 2018; Al-Kaabi et al. 2015; Kplawole 2008). These models aim to capture the complex interactions between the reservoir and aquifer, considering various factors such as fluid flow, reservoir heterogeneity capillary pressure, and relative permeability. Under saturated reservoirs are characterized by reservoir pressure above the bubble point of the reservoir fluids (Tarek 2010). This implies that the reservoir fluids are in a single phase at reservoir conditions. The saturation pressure is above the bubble point pressure, resulting in the under saturation of the reservoir. Understanding the behavior of under saturated reservoirs is crucial for effective reservoir management and optimization. Modeling these reservoirs presents unique challenges due to the presence of aquifer. The interaction between the aquifer and the reservoir affects reservoir performance and hydrocarbon recovery. The dynamic behavior of the aquifer, such as water influx and pressure support, needs to be accurately captured in the reservoir model (Ahmadreza 2018). Additionally, the uncertainty in fluid properties, such as relative permeability and capillary pressure further complicates the modeling process (Li et al. 2015).

Reservoir Characterization

Accurate reservoir characterization is essential for modeling under saturated reservoirs. This involves gathering data on reservoir properties, such as porosity, permeability, and fluid saturation. Various techniques, including well log analysis, core analysis, and seismic interpretation, are employed to characterize the reservoir (Kolawole 2008). Special attention should be given to understanding the distribution and behavior of the active aquifer. Reservoir characterization plays a vital role in understanding the behavior of under saturated reservoirs bounded by aquifers. This involves gathering data on reservoir properties, such as porosity, permeability, and fluid saturation. Well log analysis, core analysis, and seismic interpretation techniques are commonly used to characterize the reservoir. Special attention should be given to understanding the distribution and behavior of the active aquifer (Brantson et al. 2018)

Fluid Flow Model. Fluid flow modeling in under saturated reservoirs requires the incorporation of both the reservoir and aquifer dynamics. Numerical simulation models, such as black oil or compositional models are commonly used to simulate fluid flow and predict reservoir performance (Ezuka, et al. 2004). These models should consider the interaction between the reservoir and the aquifer, including water influx and pressure support mechanisms.

Models for Under Saturated Reservoirs. Numerous models are available in the literature that captures the behavior of under saturated reservoir that is bounded by aquifer (Patricia 2018). The Buckley-Leverett (1942) model is a widely used theoretical approach for describing immiscible displacement in porous media. It assumes that the fluid phases are immiscible and that capillary pressure is the primary driving force for fluid flow. The model is based on the conservation equations for mass and momentum. The solution methods for the Buckley-Leverett (1942) model involve numerical techniques such as finite difference or finite element methods. These methods discretize the reservoir and aquifer domains into a grid of cells, allowing for the solution of partial differential equations. The solutions can be obtained by iteratively solving the equations until convergence is achieved (Peaceman 1993). The Brooks-Corey model is another mathematical model commonly used to describe fluid flow under saturated reservoirs bounded by active aquifers. This model considers the capillary pressure-saturation relationship and the relative permeability-saturation relationship (Eric et al. 2018). The solution methods for the Brooks-Corey model also involve numerical techniques such as finite difference or finite element methods. These methods discretize the reservoir and aquifer domains into a grid of cells, allowing for the solution of partial differential equations. The solutions can be obtained by iteratively solving the equations until convergence is achieved. Another theoretical approach involves the use of numerical simulation techniques, such as finite difference or finite element methods. These methods discretize the reservoir and aquifer domains into a grid of cells, allowing for the solution of partial differential equations that describe fluid flow and transport processes (Brantson et al. 2018). Various numerical simulators, such as Eclipse and CMG, have been developed to simulate the behavior of under saturated reservoirs bounded by active aquifers (Aziz and Settari 1979).

Empirical studies play a crucial role in validating and calibrating theoretical models. These studies involve field data analysis, laboratory experiments, and case studies from actual reservoirs (Duretta et al. 2016). They provide valuable insights into the behavior of under saturated reservoirs bounded by active aquifers and help refine the theoretical models. Field data analysis involves the collection and analysis of production and pressure data from actual reservoirs. By comparing the observed data with the predictions of theoretical models, researchers can assess the accuracy and reliability of the models. Field data analysis has been conducted in various reservoirs, such as the Permian Basin in the United States and the Ghawar Field in Saudi Arabia (Al-Kaabi et al. 2015). Laboratory experiments are conducted to investigate specific aspects of fluid flow and transport in under saturated reservoirs bound by active aquifers. These experiments often involve core flooding tests, where rock samples are saturated with fluids and subjected to different pressure and flow conditions (Chen et al. 2006). The results of laboratory experiments provide valuable data for model calibration and validation (Kazemi et al. 1976). Case studies from actual reservoirs provide real-world examples of the behavior of under saturated reservoirs

bound by active aquifers. These case studies often involve detailed reservoir characterization, history matching, and production forecasting. They allow researchers to assess the applicability and limitations of theoretical models in practical reservoir engineering (Lake et al. 1989). In addition to mathematical models, numerical simulation techniques are commonly used to simulate the behavior of under saturated reservoirs bound by active aquifers. Two widely used numerical simulation techniques are the finite difference method and the finite element method (Azizi and Settari 1979).

Finite Difference Method. The finite difference method discretizes the reservoir and aquifer domains into a grid of cells and approximates the derivatives in the governing equations using finite differences. The governing equations, such as the conservation of mass and momentum equations, are then solved iteratively using numerical algorithms. The solutions are obtained by updating the values of the variables at each grid cell until convergence is achieved. Sample Solution for the Finite Difference Method is the explicit method, where the values of the variables at the next step are calculated based on the values at the current time step. This method is relatively simple and computationally efficient, but it has stability limitations for certain types of problems. Another solution method is the implicit method, where the values of the variables at the next time step are calculated based on a system of equations that involve the values at both the current and next time steps. This method is more computationally intensive but can handle a wider range of problems (Ahmed and McKinney 2015)

Mathematical Model. To describe the physics of primary depletion in an under saturated reservoir, we rely on fundamental governing laws, specifically conservation of mass, Darcy's law, and the equation of state. Here, we assume that the reservoir pressure remains above the bubble point pressure, rendering it saturated, with negligible mass transfer between phases. The general flow equation for the reservoir is developed by integrating the principles of mass conservation, Darcy's law, and the equation of state. For one-dimensional flow within a control volume (as shown in Figure 1), the mass balance is expressed as:

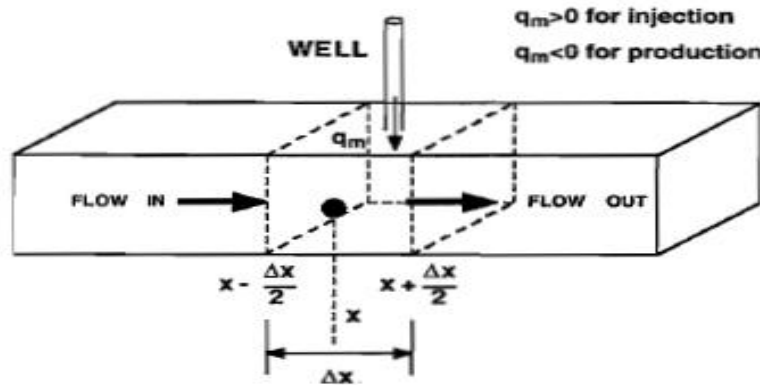


Figure 1—Control volume for 1D flow rectangular coordinate.

$$(m_i - m_o)_c + (m_s)_c + m_{aq} = (m_a)_c, \dots \dots \dots (1)$$

The three-dimensional extension of Equation (1) is formulated as:

$$\frac{d}{dx} \left[\beta_c \frac{k_x}{\mu} A_x \left(\frac{dP}{dx} - \gamma_c \rho g \frac{dz}{dx} \right) \right] \Delta x + \frac{d}{dy} \left[\beta_c \frac{k_y}{\mu} A_y \left(\frac{dP}{dy} - \gamma_c \rho g \frac{dz}{dy} \right) \right] \Delta y + \frac{d}{dz} \left[\beta_c \frac{k_z}{\mu} A_z \left(\frac{dP}{dz} - \gamma_c \rho g \frac{dz}{dz} \right) \right] \Delta z + B^o q_{s.c} + q_a = \frac{V_b \phi}{\alpha_c} \frac{dP}{dt}, \dots \dots \dots (2)$$

Rearranging yields

$$\begin{aligned} & \frac{d}{dx} \left(\beta_c \frac{k_x}{\mu} A_x \frac{dP}{dx} \right) \Delta x + \frac{d}{dy} \left(\beta_c \frac{k_y}{\mu} A_y \frac{dP}{dy} \right) \Delta y + \frac{d}{dz} \left(\beta_c \frac{k_z}{\mu} A_z \frac{dP}{dz} \right) \Delta z + B^o q_{s.c} + q_a = \frac{V_b \phi_c}{\alpha_c} \frac{dP}{dt} + \\ & \frac{d}{dx} \left(\beta_c \frac{k_x}{\mu} A_x \gamma_c \rho g \frac{dZ}{dx} \right) \Delta x + \frac{d}{dy} \left(\beta_c \frac{k_y}{\mu} A_y \gamma_c \rho g \frac{dZ}{dy} \right) \Delta y + \frac{d}{dz} \left(\beta_c \frac{k_z}{\mu} A_z \gamma_c \rho g \frac{dZ}{dz} \right) \Delta z, \dots \dots \dots (3) \end{aligned}$$

Discretization of the Flow Equation. The finite difference approximation of Equation (3) uses the forward central difference, resulting in:

$$\begin{aligned} & A_{i+1,j,k} P_{i+1,j,k}^{n+1} + B_{i,j,k} P_{i,j,k}^{n+1} + C_{i-1,j,k} P_{i-1,j,k}^{n+1} + D_{i,j+1,k} P_{i,j+1,k}^{n+1} + E_{i,j-1,k} P_{i,j-1,k}^{n+1} + F_{i,j,k+1} P_{i,j,k+1}^{n+1} + \\ & G_{i,j,k-1} P_{i,j,k-1}^{n+1} = - \left[\frac{1}{\alpha} P_{i,j,k}^n + B^o (q_{s.c})_{i,j,k} \right] + S_{G_{i,j,k}} + (q_w)_{i,j,k}, \dots \dots \dots (4) \end{aligned}$$

This represents the implicit form for slightly compressible flow in three dimensions, where:

$$\begin{aligned} S_{G_{i,j,k}} = & A_{G_{i+1,j,k}} Z_{i+1,j,k} + B_{G_{i,j,k}} Z_{i,j,k} + C_{G_{i-1,j,k}} Z_{i-1,j,k} + D_{G_{i,j+1,k}} Z_{i,j+1,k} + E_{G_{i,j-1,k}} Z_{i,j-1,k} + F_{G_{i,j,k+1}} Z_{i,j,k+1} + \\ & G_{G_{i,j,k-1}} Z_{i,j,k-1}, \dots \dots \dots (5) \end{aligned}$$

$$B_{G_{i,j,k}} = - \left(A_{G_{i+1,j,k}} + C_{G_{i-1,j,k}} + D_{G_{i,j+1,k}} + E_{G_{i,j-1,k}} + F_{G_{i,j,k+1}} + G_{G_{i,j,k-1}} \right), \dots \dots \dots (6)$$

$$B_{i,j,k} = - \left(\frac{1}{\alpha} + A_{i+1,j,k} + C_{i-1,j,k} + D_{i,j+1,k} + E_{i,j-1,k} + F_{i,j,k+1} + G_{i,j,k-1} \right), \dots \dots \dots (7)$$

Well Model. For the well model, the oil flow rate $(q_{s.c})_{i,j,k}$ is expressed in terms of productivity index for a specified bottom-hole pressure. The inner boundary condition is:

$$(q_{s.c})_{i,j,k} = -J_w (P_{i,j,k} - P_{wf}), \dots \dots \dots (8)$$

where the well block productivity index is given by:

$$J_w = - \frac{G_w}{\mu B}, \dots \dots \dots (9)$$

Aquifer Model. The unsteady-state aquifer model developed by Fetkovich (1971) begins with a productivity index equation,

$$\frac{dw_e}{dt} = J(\bar{p}_a - p), \dots \dots \dots (10)$$

Applying finite difference notation, the aquifer influx at grid blocks adjacent to the aquifer-reservoir interface becomes,

$$(q_w)_{i,j,k} = J(\bar{p}_a - p_{i,j,k}), \dots \dots \dots (11)$$

After incorporating a simple aquifer equation and integrating, we have,

$$\frac{dw_e}{dt} = J(p_i - p) e^{-Jp_i t / w_{ei}} \dots \dots \dots (12)$$

In three-dimensional finite difference form, the aquifer influx at grid blocks adjacent to the reservoir-aquifer interface is expressed as:

$$(q_w)_{i,j,k} = J(p_i - p_{i,j,k})e^{-Jp_i t/w_{ei}} \dots\dots\dots(13)$$

The maximum water influx is g W_{ei} given by

$$W_{ei} = c_t W_i p_i f \dots\dots\dots (14)$$

where

$$W_i = \left[\frac{\pi(r_a^2 - r_e^2)h\phi}{5.615} \right] \dots\dots\dots (15)$$

The boundary conditions are the aquifer outer boundaries is expressed as,

$$\bar{p}_a = p_{i,j,k} + (p_i - p_{i,j,k})e^{-Jp_i t/w_{ei}} \dots\dots\dots (16)$$

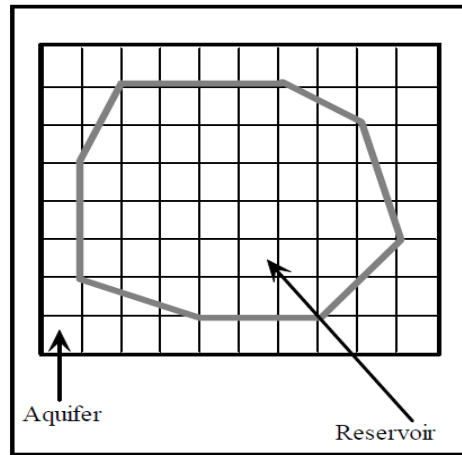


Figure 2—Analytical aquifer model with the reservoir.

Coupling Reservoir Model, Well Model and Aquifer Model. The reservoir model (Eq. 4), well model (Eq. 8), and aquifer model (Eq. 13) are coupled by substituting the well and aquifer expressions into the flow equation. Simplifying, we obtain

$$A_{i+1,j,k}P_{i+1,j,k}^{n+1} + \left((B_i - B^0 J_o + J_{aq} e^{-\frac{J_{aq} P_i^n \Delta t}{w_{ei}}}) P_{i,j,k}^{n+1} + C_{i-1,j,k} P_{i-1,j,k}^{n+1} + D_{i,j+1,k} P_{i,j+1,k}^{n+1} + E_{i,j-1,k} P_{i,j-1,k}^{n+1} + F_{i,j,k+1} P_{i,j,k+1}^{n+1} + G_{i,j,k-1} P_{i,j,k-1}^{n+1} \right) = - \left[\frac{1}{a} P_{i,j,k}^n + B^0 J_o P_{wf} + J_{aq} e^{-\frac{J_{aq} P_i^n \Delta t}{w_{ei}}} P_{i,j,k}^n \right] + S_{G_{i,j,k}} \dots\dots\dots(17)$$

Results and Discussion

The reservoir is discretized into small grid blocks, and the reservoir-aquifer flow equation is incorporated at each grid cell. The simulation study is conducted within the MATLAB environment, utilizing both MATLAB's Graphical User Interface (GUI) and MATLAB programming language. The reservoir properties, aquifer characteristics, and well parameters used in the simulation are provided in **Table 1**, with all parameters entered through the MATLAB GUI. The initial oil in place, calculated volumetrically, is 1.82×10^7 lbb.

Table 1—Reservoir and aquifer properties

Reservoir and aquifer properties	Data values
Oil viscosity, μ_o	0.4 cp
Oil compressibility, C_o	1.23×10^{-5} psi ⁻¹
Rock compressibility, C_t	6.5×10^{-6} psi ⁻¹
Porosity, ϕ	0.25
Water saturation, S_w	0.05
Reservoir bulk volume, V_b	6.4×10^6 bbl
Formation volume factor, B_o	1.2851
Initial reservoir pressure, p_i	6000 psi
Well radius, r_w	7 inches
Reservoir length, L_x	3000 ft
Reservoir width, L_y	3000 ft
Reservoir thickness, H	40 ft
Wellbore productivity index, J_w	3.0 bbl/day/psi
Aquifer productivity index, J_{aq}	120 bbl/day/psi
Maximum water influx, W_{ei}	150×10^6 bbl
Specified bottom-hole pressure, P_{wf}	5200 psi
Permeability in X direction, k_x	150 md
Permeability in Y direction, k_y	200 md
Permeability in Z direction, k_z	50 md
Bubble point pressure, p_b	2500

Discretization of the Reservoir. The dimensions of the reservoir used in the simulation are 3000 ft×3000 ft×40 ft. The reservoir is partitioned into small grid blocks using an 11×11×11 grid, resulting in a total of 1331 grid blocks for the simulation. Each grid block is unique, with varying permeability and porosity values, representing a heterogeneous formation. The aquifer interacts with the edges of the reservoir, while the top and bottom of the reservoir are modeled as no-flow boundaries. Figure 3 illustrates the discretization of the reservoir and the boundary conditions. A vertical wellbore is drilled at the center of the reservoir, passing through grid blocks (6, 6, 4), (6, 6, 5), (6, 6, 6), (6, 6, 7), (6, 6, 8), (6, 6, 9), (6, 6, 10), and (6, 6, 11), corresponding to grid block numbers (424), (545), (666), (787), (908), (1029), (1150), and (1271). The wellbore is perforated in grid blocks (6, 6, 6), (6, 6, 7), (6, 6, 8), and (6, 6, 9). This positioning ensures efficient oil flow, with the aquifer assisting at the edges of the reservoir. The total length of the drilled wellbore is 29 ft, with the perforated length being 14.56 ft. Each of the 1331 grid blocks measures 275.72 ft × 275.72 ft × 3.64 ft.

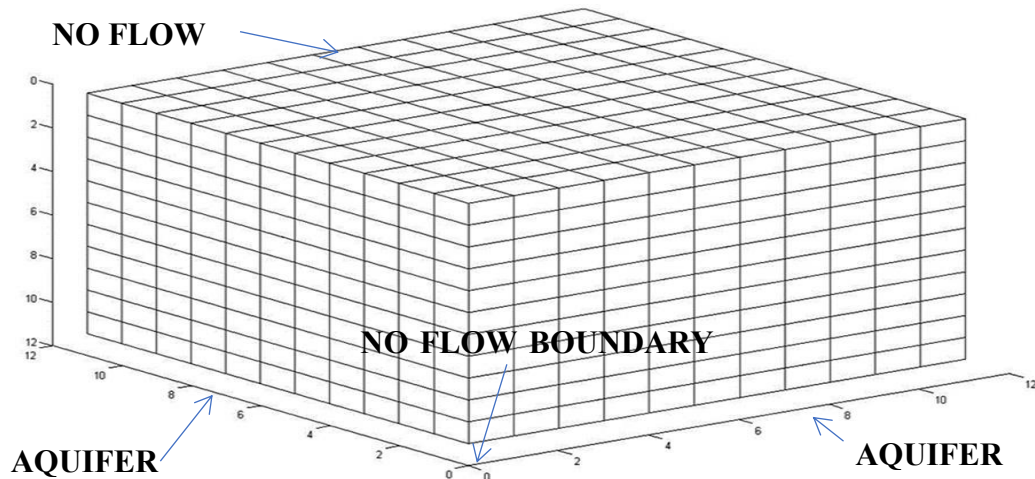


Figure 3—Schematic diagram of three dimensional (11*11*11) discretized reservoir grid system.

Average Reservoir Pressure Analysis. Figure 4 shows the average reservoir pressure and the corresponding contour map for the 1331 grid blocks, as the reservoir is depleted over time. The average reservoir pressure, calculated at each time step, varies depending on the time step size used in the computation. To minimize truncation errors, smaller time steps and mesh sizes were employed. A smaller time step results in more accurate predictions compared to larger time steps. The pressure transient caused by fluid withdrawal from the vertical well is most pronounced near the center of the reservoir, with a noticeable pressure drop at the well location. In contrast, the pressure at the boundary of the reservoir remains elevated, indicating that the boundaries are being recharged by the aquifer influx into the grid blocks.

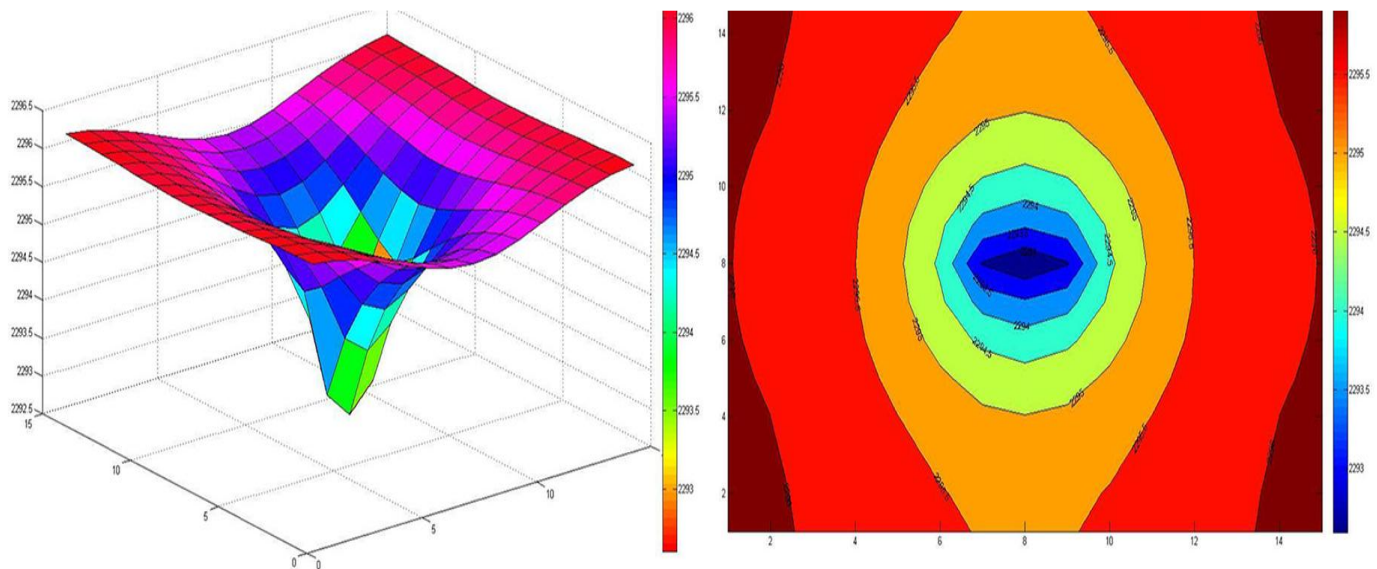


Figure 4—Surface and contour pressure of the 11*11*11 grid block.

Well block Pressure and bottom-hole versus time. Figure 5 presents the well block pressure and bottom-hole pressure as functions of time. It is observed that the bottom-hole pressure remains constant at 5200 psi throughout the production period. Conversely, the well block pressure decreases over time, reflecting the unsteady state induced by oil production. The well stops flowing once the abandonment pressure is reached, which is

approximately equal to the bottom-hole flowing pressure (5200 psi). At this point, the primary energy of the reservoir is depleted. However, as the reservoir is bound by an active aquifer, production is sustained for a longer duration.

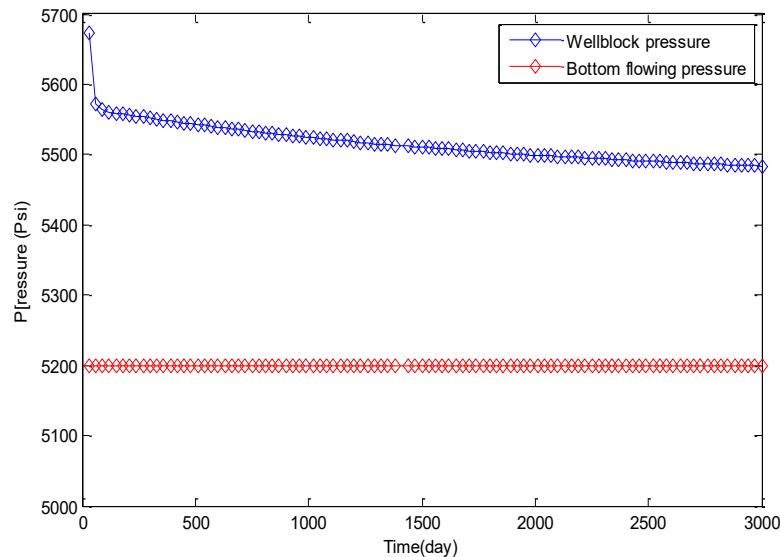


Figure 5—Well block pressure, bottom- hole pressure versus time.

Oil Production Rate and Aquifer Influx Rate Versus Time. Figure 6 depicts the relationship between the oil production rate and the aquifer influx rate over time. As oil production decreases, the aquifer influx increases, which is attributed to the growing pressure differential at the aquifer-reservoir, interface due to oil withdrawal. This highlights the crucial role of the aquifer in supporting reservoir pressure and maintaining production over time. Furthermore, the oil production rate can be estimated at any given time using the data from the plot.

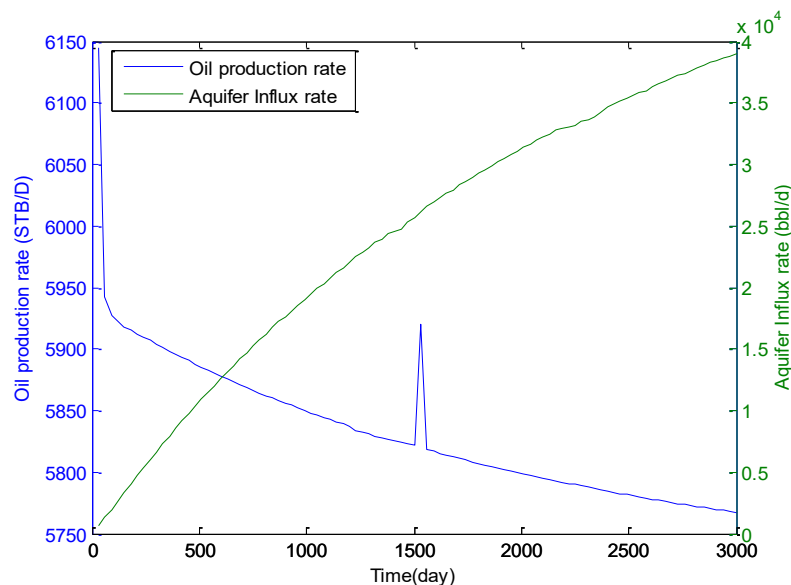


Figure 6—Oil production rate and aquifer influx versus time.

Oil Production versus Time. Figure 7 illustrates the decline in oil production over time. Notably, the production extends well beyond 3000 days, due to the support provided by the aquifer at the reservoir boundaries. At 2500 days, for instance, the oil production is estimated to be 1.74×10^5 bbl. This extended production duration enables better forecasting of the reservoir's future performance and estimation of remaining reserves.

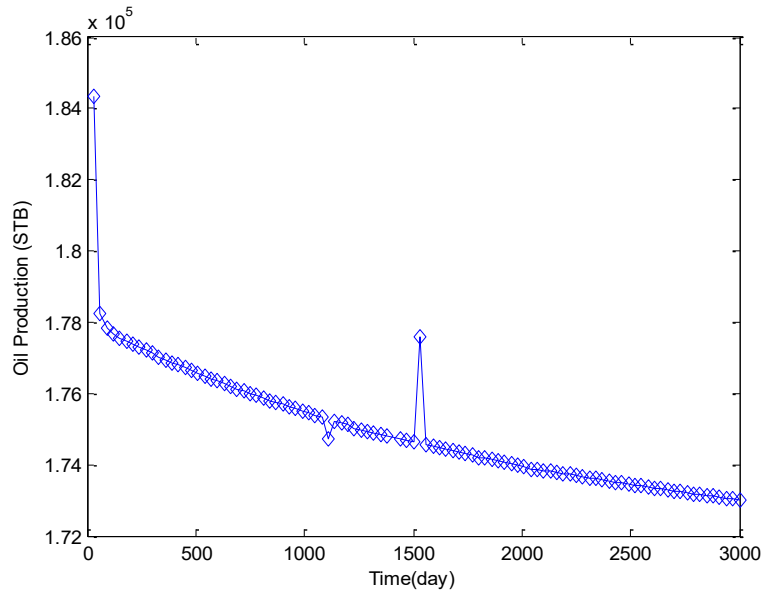


Figure 7—Oil production versus time.

Cumulative Oil Production Versus Time. Figure 8 shows the total cumulative oil production from the reservoir as a function of time. This curve represents the total amount of oil recovered from the reservoir over the production period. At 3000 days, the total cumulative oil production is approximately 1.732×10^6 bbl. This cumulative production curve is useful for evaluating the overall recovery efficiency and estimating the future recovery potential of the reservoir.

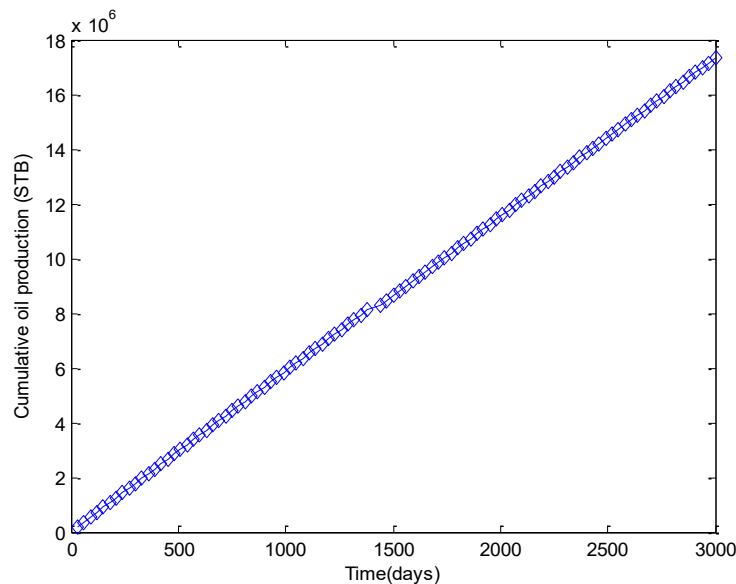


Figure 8—Total cumulative oil production versus time.

Conclusions

This project developed a three-dimensional reservoir-aquifer model for an under saturated oil reservoir, utilizing the governing equations that describe the physical processes occurring during primary production. The simulation was executed by progressing from the initial reservoir conditions to future time steps. The following conclusions can be drawn from this work. The developed simulator integrates various governing laws and reservoir simulation techniques, providing a comprehensive tool for analyzing the reservoir-aquifer system during primary recovery. The simulator can estimate the reserves of an under saturated reservoir bounded by an aquifer, specifically for reservoirs producing under primary recovery conditions.

1. The validity of the simulator is contingent on the reservoir pressure remaining above the oil bubble point pressure throughout the production process.
2. The simulator can be utilized to predict the future performance of the reservoir-aquifer system under primary recovery, facilitating long-term production planning.
3. Sensitivity analysis can be conducted using the numerical simulator to optimize reservoir parameters, enhancing recovery efficiency under various operating conditions.
4. The material balance error check provides a means of validating the numerical simulator, ensuring its reliability in predicting primary recovery until the bubble point pressure is reached.

Overall, the simulator developed in this project represents a robust tool for analyzing and predicting the behavior of under saturated oil reservoirs with aquifer support during primary recovery, aiding in decision-making for reservoir management and future production optimization.

Conflicting Interests

The author(s) declare that they have no conflicting interests.

Nomenclature

γ_c	=	conversion factor, 0.215884×10^{-3}
g	=	acceleration due to gravity, 32.174 ft/sec^2
ρ	=	density, lbm/ft^3
v	=	velocity of the moving fluid, ft/sec
β_c	=	conversion factor, 1.127
k	=	absolute permeability, md
μ	=	fluid viscosity, cp
m_i	=	mass in or mass of the component entering the control volume from other parts of the reservoir, kg/s
m_o	=	mass out or mass of the component leaving the control volume to other part of the reservoir, kg/s
m_s	=	source/sink or mass of the components leaving or entering the control volume externally through the well. For source (production) is negative, while for sink (injection) is positive, kg/s
m_a	=	mass accumulated or mass of excess material stored in or depleted from the control volume over a time interval, kg/s
e_w	=	rate of water influx, bbl/day
k	=	permeability of the aquifer, md
h	=	thickness of the aquifer, ft
r_a	=	radius of the aquifer, ft
r_e	=	radius of the reservoir
t	=	time, days
$\bar{p}_a, p_{i,j,k}$	=	average aquifer pressure and aquifer-reservoir boundary pressure, psia

W_i, W_{ei}	=	initial volume of water in the aquifer and maximum aquifer water influx, bbl
r_e, r_a	=	reservoir and aquifer radius, ft
$(q_w)_{i,j,k}$	=	water influx rate, bbl/day
J	=	aquifer productivity index, bbl/day/psi
c_t	=	total compressibility, psi^{-1}
h	=	aquifer thickness, ft
$\emptyset$	=	aquifer porosity, dimensionless
J_w	=	well block productivity index, STB/D-psi
G_w	=	well block geometric factor, RB-cp/D-psi
P_{wf}	=	flow bottom-hole pressure, psi

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